

# Adjustment of Adaptive Gain with Bounded Linear Stability Analysis to Improve Time-Delay Margin for Metrics-Driven Adaptive Control

Maryam Bakhtiari-Nejad\*

Nhan T. Nguyen<sup>†</sup>

Kalmanje Krishnakumar<sup>‡</sup>

*NASA Ames Research Center, Moffett Field, CA 94035*

**This paper presents the application of Bounded Linear Stability Analysis (BLSA) method for metrics-driven adaptive control. The bounded linear stability analysis method is used for analyzing stability of adaptive control models, without linearizing the adaptive laws. Metrics-driven adaptive control introduces a notion that adaptation should be driven by some stability metrics to achieve robustness. By the application of bounded linear stability analysis method the adaptive gain is adjusted during the adaptation in order to meet certain phase margin requirements. Analysis of metrics-driven adaptive control is evaluated for a linear damaged twin-engine generic transport model of aircraft. The analysis shows that the system with the adjusted adaptive gain becomes more robust to unmodeled dynamics or time delay.**

## I. Introduction

Various adaptive flight control techniques have been investigated over the past number of years.<sup>1-9</sup> Adaptive control system adapts to system uncertainty to maintain performance and stability. However, there still remain challenges in obtaining system robustness in the presence of unmodeled dynamics, parameter uncertainties, and disturbances. Adaptive control laws are generally nonlinear and therefore stability robustness of adaptive control cannot be analyzed by linear stability metrics in terms of phase and gain margins. These margins are designed into linear control laws to provide robustness to account for system uncertainties such as modeling errors and exogenous disturbances. The lack of stability metrics for adaptive control is a major challenge to enable adaptive control laws to be adopted in production control systems. Metrics-driven adaptive control introduces a notion that adaptation should be driven by some stability metrics to achieve robustness.<sup>10</sup> The bounded linear stability analysis method is applied in order to analyze adaptive control in terms of the linear stability concept by establishing an approximate linear equivalent system as a function of a mean square value of the input function to the adaptive law.<sup>12</sup> The method uses an error bound analysis to extract the dominant linear components of the nonlinear adaptive laws without linearization. The idea is to seek a linear representation that bounds a nonlinear adaptive law.<sup>12</sup> This linear equivalent system is only used for analysis and not for actual adaptation. This method can provide an understanding of the stability margin of nonlinear adaptive control that can be used to establish limits on adaptive gains during adaptation to ensure system robustness, and thus the adaptation is made to be metrics-driven. The bounded linear stability analysis is studied in a framework of a model reference adaptive control to improve system robustness.

## II. Bounded Linear Stability Analysis (BLSA)

Given a plant model as<sup>11</sup>

$$\dot{x} = A_p x + B_p u + f(x) \quad (1)$$

where  $x \in \mathbb{R}^n$  is a state vector,  $u \in \mathbb{R}^n$  is a control vector, and  $A_p, B_p \in \mathbb{R}^{n \times n}$  are known plant matrices, and  $f(x) \in \mathbb{R}^n$  is an unmatched, bounded uncertainty.

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\*Aerospace Engineer, Intelligent Systems Division, Mail Stop 269-1, AIAA Member

<sup>†</sup>Research Scientist, Intelligent Systems Division, Mail Stop 269-1, AIAA Associate Fellow

<sup>‡</sup>Research Scientist, Intelligent Systems Division, Mail Stop 269-1, AIAA Associate Fellow

The objective is to produce a controller that enables the plant to follow a reference model described by

$$\dot{x}_m = A_m x_m + B_m r \quad (2)$$

where  $A_m \in \mathbb{R}^{n \times n}$  is Hurwitz and given,  $B_m \in \mathbb{R}^{n \times n}$  is also given, and  $r \in \mathbb{R}^n \in \mathcal{L}_\infty$  is a bounded command vector with  $\dot{r} \in \mathbb{R}^n \in \mathcal{L}_\infty$  also bounded.

Defining the tracking error as  $\tilde{x} = x_m - x$ , the goal is to determine a controller that results in  $\lim_{t \rightarrow \infty} \|\tilde{x}\| = 0$ . A dynamic inversion controller is designed to give the tracking error a second-order response with a proportional-integral feedback control as

$$u = B_p^{-1} \left( \dot{x}_m - A_p x + K_p \tilde{x} + K_i \int_0^t \tilde{x} d\tau - u_{ad} \right) \quad (3)$$

where  $K_p = \text{diag}(k_{p,1}, \dots, k_{p,n}) > 0$ , and  $K_i = \text{diag}(k_{i,1}, \dots, k_{i,n}) > 0$ .

The tracking error dynamics are expressed as

$$\dot{\tilde{x}} = -K_p \tilde{x} - K_i \int_0^t \tilde{x} d\tau + u_{ad} - f(x) \quad (4)$$

Let  $e = \left[ \int_0^t \tilde{x} d\tau \quad \tilde{x} \right]^\top \in \mathbb{R}^{2n}$ , then

$$\dot{e} = A_c e + b(u_{ad} - f) \quad (5)$$

where

$$A_c = \begin{bmatrix} 0 & I \\ -K_i & -K_p \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad (6)$$

The direct adaptive signal is parameterized by a linear-in-parameter matched uncertainty as<sup>4</sup>

$$u_{ad} = W^\top \beta(x) \quad (7)$$

where  $W \in \mathbb{R}^{m \times n}$  is a weight matrix and  $\beta \in \mathbb{R}^m$  is a basis vector with Lipschitz properties

$$\|\beta(x) - \beta(x_0)\| \leq C \|x - x_0\| \quad (8)$$

for some constant  $C > 0$ , which implies a bounded derivative

$$\left\| \frac{\partial \beta(x)}{\partial x} \right\| \leq L \quad (9)$$

for some constant  $L > 0$ .

The adaptive law is then given by<sup>4</sup>

$$\dot{W} = -\Gamma \beta e^\top P b \quad (10)$$

where  $\Gamma > 0 \in \mathbb{R}$  is an adaptive gain and  $P > 0 \in \mathbb{R}^{2n \times 2n}$  solves the Lyapunov equation

$$P A_c + A_c^\top P = -Q \quad (11)$$

where  $Q > 0$  is a symmetric positive-definite matrix.

Stability of nonlinear adaptive control is usually analyzed by the Lyapunov method. The traditional linear stability margin concept may be extended to nonlinear adaptive control if it could be represented by some linear approximations. To obtain an equivalent linear time invariant system (LTI), the adaptive law can be linearized at a certain point in time when the weights are at a steady state, usually long after initial transients have settled down. However, transient responses during adaptation can be important and the adaptive law should be designed in a way that would prevent large initial transients, which can compromise system robustness. The bounded linear stability analysis seeks a piecewise linear equivalent approximation of nonlinear adaptive control over a short time window during which the LTI concept of stability margins could be analyzed to provide a method for adjusting the adaptive gain for the next time window. The linear equivalent approximation is not a replacement of an adaptive law but rather is used in conjunction with the adaptive law for the stability analysis purpose.

The following propositions and statements are cited according to Nguyen et. al.<sup>12</sup> The equilibrium state  $y = 0$  of the differential equation

$$\dot{y} = -\beta^\top(t) \Gamma \beta(t) y \quad (12)$$

where  $y(t) : [0, \infty) \rightarrow \mathbb{R}$ ,  $\beta(t) \in \mathcal{L}_2 : [0, \infty) \rightarrow \mathbb{R}^n$  is a piecewise continuous and bounded function, and  $\Gamma > 0 \in \mathbb{R}^{n \times n}$ , is uniformly asymptotically stable, if there exists a constant  $\alpha_0 > 0$  such that

$$\frac{1}{T_0} \int_t^{t+T_0} \beta^\top(\tau) \Gamma \beta(\tau) d\tau \geq \Gamma \alpha_0 \quad (13)$$

which implies that  $y$  is bounded by the solution of a linear differential equation

$$\dot{z} = -\Gamma \alpha_0 z \quad (14)$$

for  $t \in [t_i, t_i + T_0]$ , where  $t_0 = 0$ ,  $t_i = t_{i-1} + T_0$ , and  $i = 1, 2, \dots, n \rightarrow \infty$ .

Another proposition states that the solution of a linear differential equation

$$\dot{y} = Ay + g(t) \quad (15)$$

where  $y(t) : [0, \infty) \rightarrow \mathbb{R}^n$ ,  $A \in \mathbb{R}^{n \times n}$  is a Hurwitz matrix, and  $g(t) : [0, \infty) \rightarrow \mathbb{R}^n \in \mathcal{L}_\infty$  is a piecewise continuous, bounded function, is asymptotically stable and semi-globally bounded from above by the solution of a differential equation<sup>12</sup>

$$\dot{z} = A(z - \alpha |A^{-1}c|) \quad (16)$$

where  $\alpha \geq 1 \in \mathbb{R}$  and  $c = \sup_t |g(t)|$ .

Therefore, the bounded linear stability analysis approximates the adaptive law and the tracking error dynamics by a piecewise linear representation as

$$\frac{d}{dt} \begin{bmatrix} e \\ \tilde{W}^\top \beta \end{bmatrix} \leq \begin{bmatrix} A_c & b \\ -\Gamma \alpha_0 b^\top P & 0 \end{bmatrix} \begin{bmatrix} e \\ \tilde{W}^\top \beta \end{bmatrix} + \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \quad (17)$$

for  $t \in [t_i, t_i + T_0]$ , where  $t_0 = 0$ ,  $t_i = t_{i-1} + T_0$ , and  $i = 1, 2, \dots, n \rightarrow \infty$ , where  $\tilde{W}$  is the weight variation, and  $\Delta_1$  and  $\Delta_2$  are some upper bounds.

The piecewise linear approximation of the nonlinear adaptive laws and the tracking error dynamics over a local time window enables the adaptive control to be analyzed in the context of an equivalent LTI system in a local sense from which system robustness can be assessed via the linear stability margin concept during that time window. The window width  $T$  can be adjusted to sufficiently capture initial transients for analyzing system robustness.

### III. Model Reference Adaptive Control

A rate-command-attitude hold (RCAH) controller is designed using a reduced-order equation of the linearized angular motion of a damaged aircraft model as<sup>13</sup>

$$\dot{\omega} = A_1 \omega + A_2 \sigma + B \delta \quad (18)$$

where  $\omega = \begin{bmatrix} p & q & r \end{bmatrix}^\top$  is the aircraft angular rate,  $\sigma = \begin{bmatrix} \alpha & \beta & \phi & \delta_r \end{bmatrix}^\top$  is a trim state vector to maintain trim condition,  $\delta = \begin{bmatrix} \delta_a & \delta_e & \delta_r \end{bmatrix}^\top$  is a control vector of aileron, elevator, and rudder deflections,  $A_1 \in \mathbb{R}^{3 \times 3}$ ,  $A_2 \in \mathbb{R}^{3 \times 4}$ , and  $B \in \mathbb{R}^{3 \times 3}$  are true plant matrices which are unknown. The nominal aircraft dynamics is described by

$$\dot{\omega}^* = A_1^* \omega + A_2^* \sigma + B^* \delta \quad (19)$$

where  $A_1^*$ ,  $A_2^*$ , and  $B^*$  are the nominal plant matrices which are assumed to be known. These matrices can generally be assumed to be associated with an ideal or undamaged aircraft.

An architecture of the model reference adaptive control (MRAC) method is shown in figure 1. This architecture uses a reference model that translates rate commands into desired acceleration commands, a proportional-integral (PI) feedback control for rate stabilization and tracking, a dynamic inversion controller that computes a control allocation using desired acceleration commands, and a neural net direct adaptive law to compensate for the tracking error.

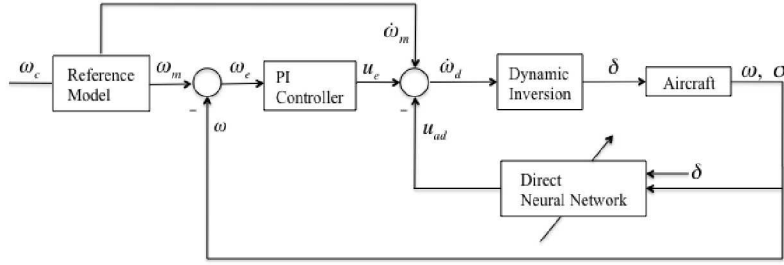


Fig. 1 - Model Reference Adaptive Control (MRAC) Architecture.

The reference model filters a pilot command  $\omega_c$  into a reference angular rate  $\omega_m$  via a first-order model

$$\dot{\omega}_d = A_m \omega_m + B_m \omega_c \quad (20)$$

where  $A_m = -\text{diag}(\omega_1, \omega_2, \omega_3) \in \mathbb{R}^{3 \times 3}$  is Hurwitz and  $B_m \in \mathbb{R}^{3 \times 3}$ . For the transport aircraft, typical values for  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$  are 3.5, 2.5, and 2.0 rad/s for roll, pitch, and yaw respectively.

A tracking error signal  $\omega_e = \omega_m - \omega$  is formed by comparing the reference angular rate with the actual angular rate output. The inner loop is then closed with a proportional-integral (PI) controller  $u_e$  operated on the tracking error signal as

$$u_e = K_p \omega_e + K_i \int_0^t \omega_e d\tau \quad (21)$$

where  $K_p \in \mathbb{R}^{n \times n}$  and  $K_i \in \mathbb{R}^{n \times n}$  are diagonal positive-definite proportional and integral gain matrices. The PI controller is designed to better handle errors detected from the angular rate feedback. A windup protection is included to limit the integrator at its current value when a control surface is saturated.

Let  $\dot{\omega}_d$  be a desired acceleration that comprises the reference model acceleration, PI controller, and a neural net adaptive signal.

$$\begin{aligned} \dot{\omega}_d &= \dot{\omega}_m + u_e - u_{ad} \\ &= A_m \omega_m + B_m \omega_c + K_p \omega_e + K_i \int_0^t \omega_e d\tau - u_{ad} \end{aligned} \quad (22)$$

The dynamic inversion controller is obtained using the nominal plant model, assuming that  $B^*$  is invertible.

$$\delta = B^{*-1} (\dot{\omega}_d - A_1^* \omega - A_2^* \sigma) \quad (23)$$

The tracking error dynamics can be expressed as

$$\dot{e} = A_c e + b (u_{ad} - \varepsilon) \quad (24)$$

where  $e = \begin{bmatrix} \int_0^t \omega_e d\tau & \omega_e \end{bmatrix}^T$ ,  $u_{ad}$  is the direct adaptive control signal, and  $A_c \in \mathbb{R}^{6 \times 6}$  and  $b \in \mathbb{R}^{6 \times 3}$  are defined as

$$A_c = \begin{bmatrix} 0 & I \\ -K_i & -K_p \end{bmatrix} > 0, \quad b = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

The adaptive signal  $u_{ad}$ , and adaptive law are defined according to Eq.7 and Eq.10 respectively.

As explained in the previous section, the stability of the adaptive law can be analyzed by a conservatively bounded linear stability using a linear equivalent adaptive law.

Let  $W^*$  be the constant ideal weight and  $\tilde{W} = W - W^*$  be the weight variation, therefore

$$\frac{d}{dt} (\tilde{W}^\top \beta) \leq -\Gamma \alpha_0 b^\top P e + \Delta_1 \quad (25)$$

where  $\Delta_1 > 0$  is a constant that represents an upper bound error, and  $\alpha_0 > 0$  is a mean square value of the input function to the adaptive law such that

$$\alpha_0 \leq \frac{1}{T} \int_t^{t+T} \beta^\top \beta d\tau \approx \frac{1}{n} \sum_{k=0}^{n-1} \beta^\top \beta (t + kT) \quad (26)$$

where  $T = n\Delta t$  is the analysis time window and  $n$  is the number of time steps  $\Delta t$ .

Since  $u_{ad} = (W^* + \tilde{W})^\top \beta$ , then the linear bound to the adaptive law (Eq. 25) is rewritten as

$$\frac{d}{dt} (u_{ad}) = -\Gamma \alpha_0 b^\top P e + \Delta_1 \quad (27)$$

or by Laplace transform

$$u_{ad}(s) = -\Gamma \alpha_0 \frac{P_{22}s + P_{12}}{s} \omega_e(s) + \Delta_2 \quad (28)$$

where  $\Delta_2$  is a constant bound.

Using the bounded linear stability analysis approach, the off-nominal open-loop transfer function between the output  $\omega(s)$  and the tracking error  $\omega_e(s)$  is obtained as<sup>11</sup>

$$G(s) = \frac{\omega}{\omega_e}(s) = \frac{k_p s^2 + (k_i + \Gamma \alpha_0 P_{22})s + \Gamma \alpha_0 P_{12}}{s^3 + (BB^{*-1}A_1^* - A_1)s^2} \times BB^{*-1} \quad (29)$$

If the adaptation is such that it results in a perfect model matching condition so that  $e \rightarrow 0$ , then the open-loop transfer function in Eq. (29) becomes an ideal transfer function:

$$G^*(s) = \frac{k_p s^2 + (k_i + \Gamma \alpha_0 P_{22})s + \Gamma \alpha_0 P_{12}}{s^3} \quad (30)$$

The off-nominal open-loop  $G(s)$  requires the knowledge of the true plant model, while the ideal open-loop  $G^*(s)$  contains all known parameters and thus is most suited in order to adjust the adaptive gain. The process is explained in more details in the next section.

The ideal open-loop transfer function  $G^*(s)$  is used in order to obtain an estimation of phase margin and time-delay margin of the generic transport model of aircraft, and in order to examine the effect of adaptive gain. It is well known that while increasing the adaptive gain  $\Gamma$  up to some point does improve the tracking performance, it causes a decrease in phase margin and time-delay margin. This effect is shown in figures 2 with the plots of phase margin and time-delay margin versus adaptive gain.

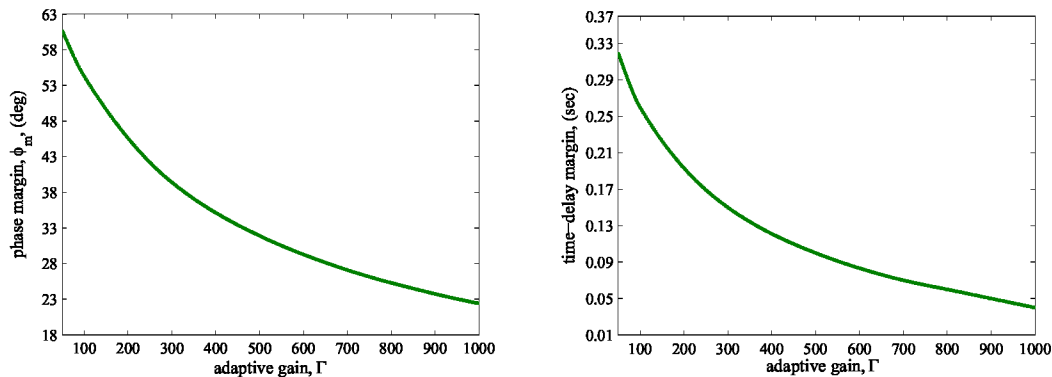


Fig. 2 - Effect of Adaptive Gain  $\Gamma$  on Phase Margin (left) and Time-delay Margin (right).

Figure 3 is a plot of the pitch rate doublet tracking, and roll and yaw rate responses for two different adaptive gains. It is shown that with  $\Gamma = 700$  an improved tracking performance is achieved as compared to performance with  $\Gamma = 50$ . However as shown in figure 2, the stability margins obtained with  $\Gamma = 50$  are up to 3 times higher. Therefore a trade-off should be made in order to choose a proper adaptive gain that both preserves the margins and provides a viable tracking performance. As discussed in the next, by adjusting the adaptive gain during adaptation in order to achieve certain criteria for phase margin, adaptation is made to be metrics-driven.

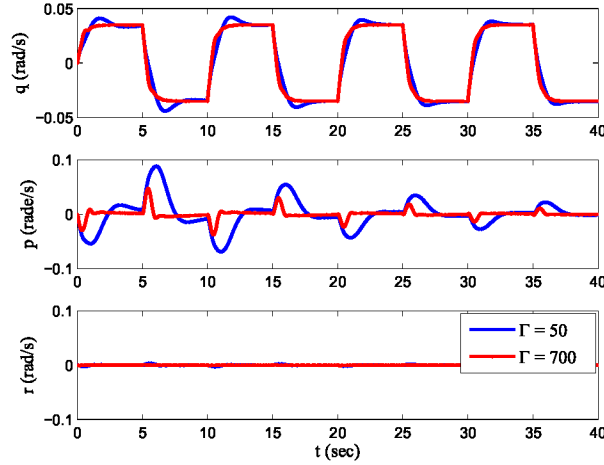


Fig. 3 - Tracking Performance to Pitch Rate Doublet Input.

#### IV. Metrics-Driven Adaptive Control

The ideal open-loop transfer function (Eq.30) contains all known parameters and therefore it could be used for adjusting the adaptive gain  $\Gamma$  during adaptation in order to meet certain phase margin requirements. The frequency response of the ideal transfer function is used to define the phase margin  $\phi_m$  and the corresponding gain crossover frequency  $\omega_g$ .

By definition, phase margin is described as  $\phi_m = \arg[G^*(j\omega_g)] + \Pi^{15-17}$  or:

$$\phi_m = \arctan\left(\frac{(k_i + \Gamma\alpha_0 P_{22})\omega_g}{\Gamma\alpha_0 P_{12} - k_p\omega_g^2}\right) - \pi/2 \quad (31)$$

and the corresponding gain-crossover frequency  $\omega_g$  is defined by  $|G^*(j\omega_g)| = 1$  or:

$$\sqrt{\frac{(\Gamma\alpha_0 P_{12} - k_p\omega_g^2)^2 + (k_i + \Gamma\alpha_0 P_{22})^2\omega_g^2}{\omega_g^4 + \omega_g^6}} = 1 \quad (32)$$

The  $\alpha_0$  parameter is computed according to Eq. (26) within a given time-window  $T$ . Using this value and a desired phase-margin  $\phi_m$ , Eqs. (31) and (32) are solved together using a nonlinear root search to calculate  $\omega_g$  and the appropriate adaptive gain  $\Gamma$ . The calculated  $\Gamma$  is then used for adaptation for the next time-window. With the purpose of reaching a desired phase-margin  $\phi_m$ , this process is repeated for each time-window. Therefore, the adjusted adaptive gain instead of a fixed gain is used in the adaptive control. The analysis result is illustrated in the simulation section.

Investigation on the convergence of the system of nonlinear equations (Eqs. 31, 32) indicates that for any desired phase margin of the range  $\phi_m \leq 65deg$ , there exist solutions for  $\Gamma$  and  $\omega_g$ . However, there is no solution for the system of nonlinear equations given the phase margins in the range  $\phi_m > 65deg$ . This fact is shown in figure 4, which demonstrates that only the graphs for the solution of phase margin Eq. (31) in the range of  $10 - 65deg$  intersect with the graph for the solution of gain crossover frequency  $\omega_g$ . No solution intersection between the phase margin  $\phi_m$  and gain crossover frequency  $\omega_g$  is obtained out of this range.

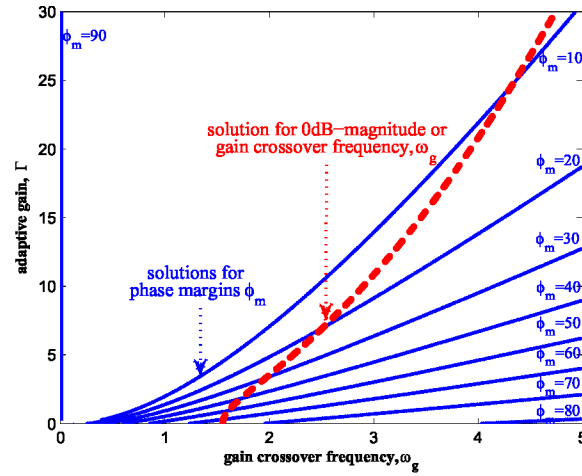


Fig. 4 - Limit on the Desired  $\phi_m$  Criteria for the Metrics-Driven Adaptive Control.

## V. Simulation

In order to illustrate metrics-driven adaptive control with the BLSA method, a simulation is performed for a damaged twin-engine generic transport model (GTM)<sup>14</sup> as shown in figure 5. A wing damage simulation is performed with 25% of the left wing missing. A pitch doublet maneuver is commanded while the roll and yaw rates are regulated.



Fig. 5 - Generic Transport Model.

As mentioned in the previous section, adjusted adaptive gain  $\Gamma$  is obtained from the analysis of the ideal open-loop transfer function, Eq. (30), with the purpose of achieving the desired phase margin  $\phi_m = 45 \deg$ . Over each time-window, the adjusted adaptive gain  $\Gamma$  is calculated, and it is then used for adaptation for the next time-window. The metrics-driven analysis as shown in this paper is only performed for the pitch axis. Figure 6 shows the  $\alpha_0$  value for the pitch axis obtained over the time-window of  $T = 0.65 \text{ sec}$  (left plot), and the corresponding adjusted adaptive gain  $\Gamma$  (right plot).

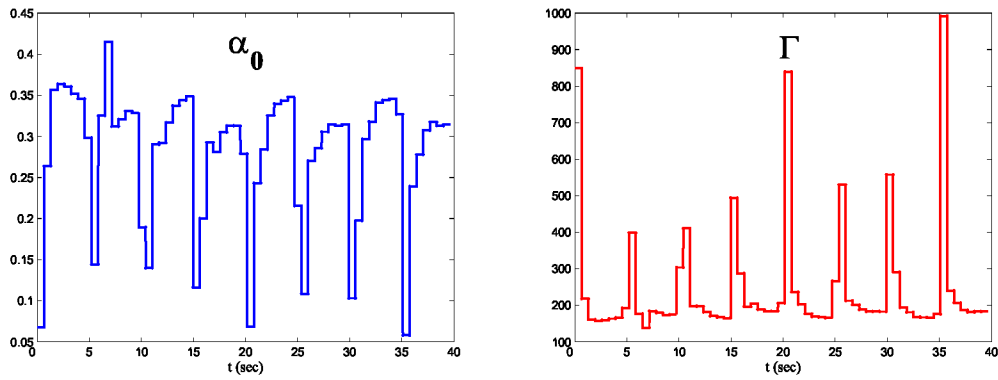


Fig. 6 -  $\alpha_0$  Parameter (left), and Adjusted Adaptive Gain  $\Gamma$  (right).

By evaluating the ideal open-loop transfer function (Eq.30) and using the adjusted adaptive gain, a phase margin estimate of  $45 \text{ deg}$  is obtained. However, if instead a fixed adaptive gain  $\Gamma = 1000$  is used, a phase margin of only  $21 \text{ deg}$  is achieved as illustrated in figure 7.

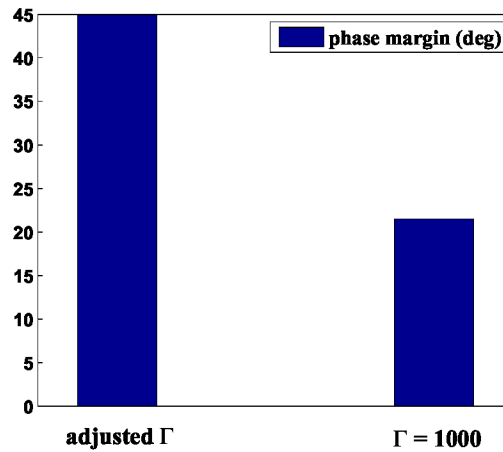


Fig. 7 - Estimated Phase Margin with Adjusted and Fixed Adaptive Gain  $\Gamma$ .

Figure 8 is a plot of the tracking performance of the aircraft angular rates to the pitch rate doublet command. The tracking performance is illustrated using both the adjusted adaptive gain and the fixed gain  $\Gamma = 1000$ . Although the pitch rate performance is not much affected by the adjusted  $\Gamma$ , the roll and yaw rate suffers with higher tracking error as compared to the performance with fixed  $\Gamma = 1000$ .

Figure 9 shows adaptive control signal  $u_{ad}$  for pitch, roll, and yaw axes. Using adjusted  $\Gamma$ , adaptive control signal is shown to become more stable, with less oscillations.



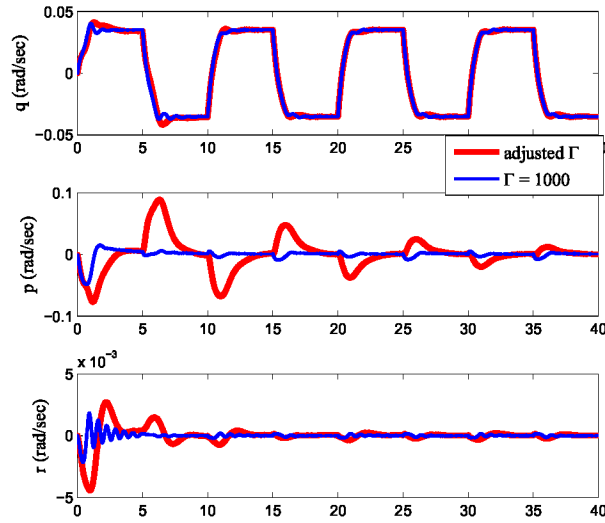


Fig. 8 - Pitch Rate Response with Adjusted and Fixed Adaptive Gains.

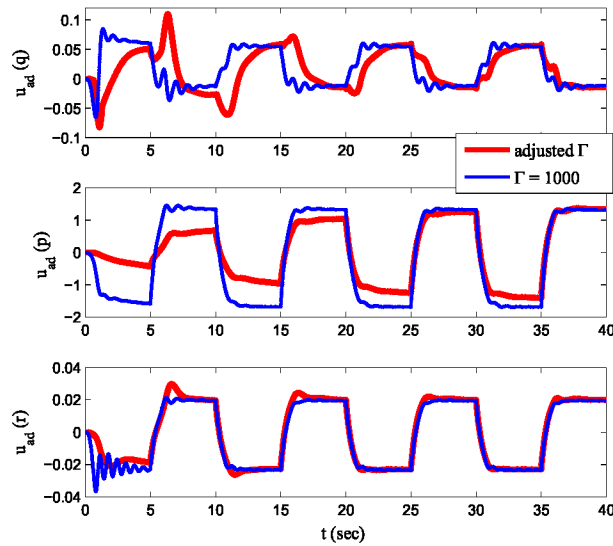


Fig. 9 - Adaptive Control Signal Response with Adjusted and Fixed  $\Gamma$ .

In order to assess the time-delay margin analysis of the metrics-driven adaptive control method, an added uncertainty as a time-delay is introduced at the control input signal to the aircraft dynamics. The maximum time-delay that system tolerates is shown in figure 10 (left plot) using adjusted and fixed  $\Gamma$ . The right plot shows the estimate of the time-delay margin obtained from the ideal open-loop transfer function (Eq.30). Both plots illustrate that with adjusted  $\Gamma$  versus fixed  $\Gamma = 1000$ , a much higher time-delay margin is achieved. This indicates that by using the adjusted  $\Gamma$ , system robustness is significantly improved. Moreover, comparing the two plots indicate that the time-delay margin analysis using BLSA method (right plot) provides a conservative and practical estimate for the actual max time-delay that system tolerates (left plot). This fact specially proves that BLSA method is reliable for the purpose of metrics-driven adaptive control.

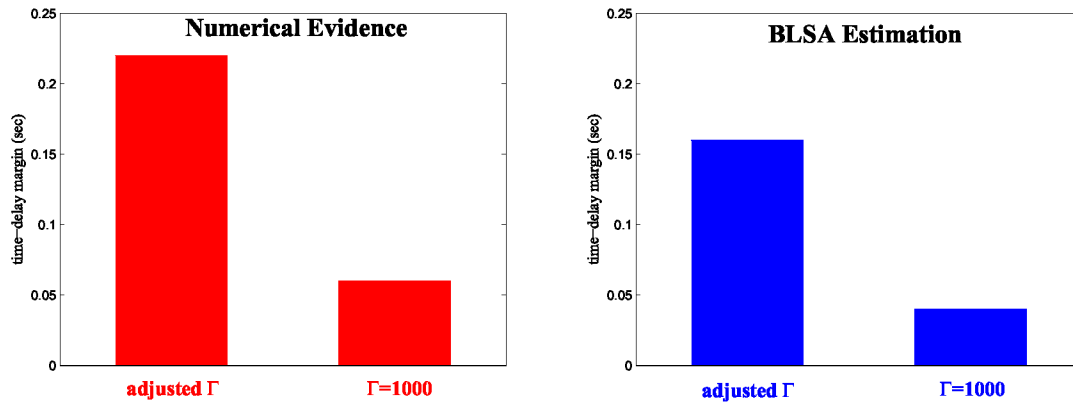


Fig. 10 - Time-Delay Analysis with Adjusted and Fixed Adaptive Gain  $\Gamma$ .

Figures 11 and 12 illustrate the angular rate performance and the adaptive control signal respectively, with the added maximum tolerated time-delay of  $t_d = 0.22\text{sec}$ . Both figures indicate that despite an added high value of time-delay  $t_d = 0.22\text{sec}$ , system still retains its stability.

Figure 13 is a plot of tracking error with added maximum time-delay. The plot again shows that with adjusted  $\Gamma$  if up to  $0.22\text{sec}$  time-delay is introduced in the system, tracking error still remains stable. While with the fixed  $\Gamma = 1000$ , system becomes unstable with only  $0.07\text{sec}$  of added time-delay.

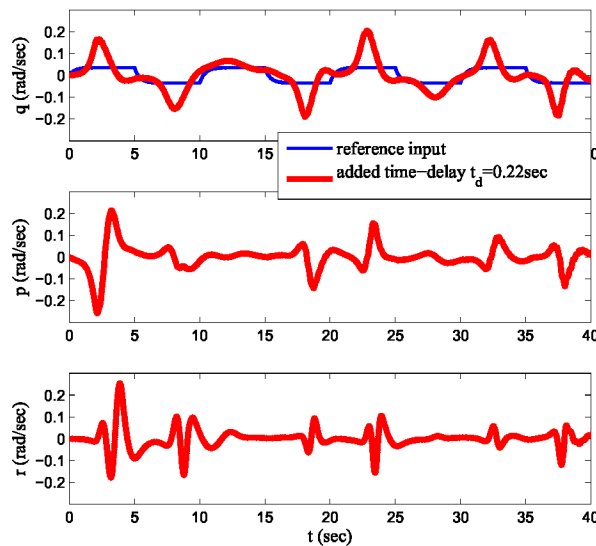


Fig. 11 - Angular Rate Tracking Performance with Added MAX. Time-Delay.

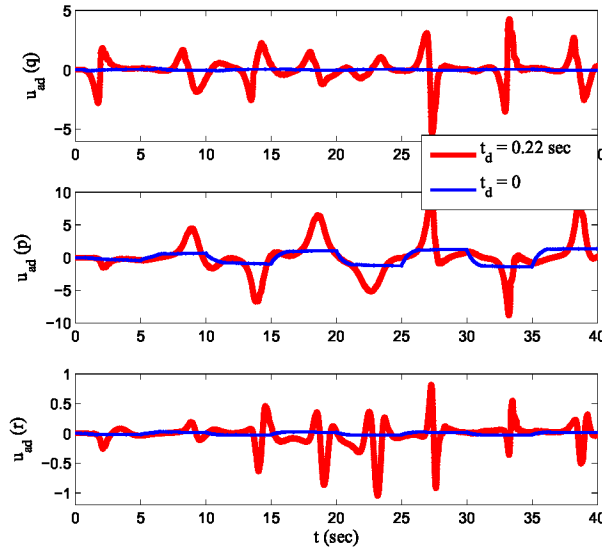


Fig. 12 - Adaptive Control Signal with Added MAX. Time-Delay.

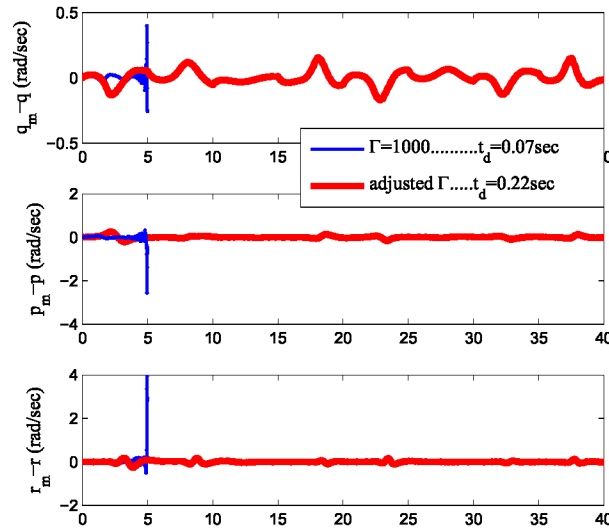


Fig. 13 - Tracking Error with Added MAX. Time-Delay.

## VI. Discussion

One of the issues with adaptive control is the lack of metrics to assess stability robustness in the presence of unmodeled dynamics or time-delay. Fast adaptation is needed in order to improve tracking performance when a system is subject to a large source of uncertainties such as structural damage to the aircraft. With fast adaptation, the direct model reference adaptive control (MRAC) results in reduced phase and time-delay margin. Therefore, adaptive gain must be chosen carefully in order to avoid instability due to time-delay or unmodeled dynamics. By adjusting the adaptive gain during adaptation to meet certain stability margin requirements, the adaptive law is made to be metrics-driven. With bounded linear stability analysis (BLSA) method, a piecewise linear upper bound for the adaptive law is formed, and from there the true off-nominal and ideal open-loop transfer functions are obtained. With the goal of achieving a certain phase margin  $\phi_m$  criteria, the ideal open-loop transfer function is used in order to adjust the adaptive

gain  $\Gamma$  during adaptation. As demonstrated in figure 4, the analysis indicates that the adaptive gain  $\Gamma$  can always be adjusted for a specific desired phase margin on the range of  $\phi_m \leq 65$  deg.

The stability margin analysis of the ideal open-loop transfer function Eq.(30) indicates that with adjusted  $\Gamma$  an estimate of the phase margin and time-delay margin of  $\phi_m = 45 \text{ deg}$  and  $TD_m = 0.16 \text{ sec}$  is obtained; while with a fixed adaptive gain  $\Gamma = 1000$ , a phase margin and time-delay margin as low as  $\phi_m = 21.5 \text{ deg}$  and  $TD_m = 0.04 \text{ sec}$  is achieved respectively (figure 7 and figure 10 \_right plot\_). Figures 8 and 9 compares the behavior of the control system with the adjusted and fixed  $\Gamma$ . The tracking performance with adjusted  $\Gamma$  is shown to suffer at the beginning of the adaptation, mostly in the roll and yaw axis (figure 8). However using the adjusted  $\Gamma$ , the adaptive control signals are shown to become even more stable as compared to an oscillatory response obtained with the fixed  $\Gamma = 1000$  (figure 9).

Time-delay analysis is performed in order to evaluate the metrics-driven adaptive control with BLSA method. By adding a time-delay between the control signal and aircraft, the simulation with adjusted  $\Gamma$  indicates that system still remains stable with as much as  $0.22 \text{ sec}$  of introduced time-delay. However with the fixed adaptive gain  $\Gamma = 1000$ , system tolerates only a very small amount  $0.07 \text{ sec}$  of time-delay (figure 10 \_left plot\_). This indicates that system with adjusted adaptive gain  $\Gamma$  becomes up to 3 times more robust to unmodeled dynamics or time-delay. Figures 11 and 12 illustrate the tracking performance and adaptive control signals with an added time-delay of  $0.22 \text{ sec}$  in the system, when adjusted  $\Gamma$  is used.

The bounded linear stability analysis in this paper is performed over a time-window of  $T = 0.65 \text{ sec}$ . It should be mentioned that a trade-off should be made between a large time-window that does not properly capture transients, and a small time-window that causes poor convergence of the adaptive parameters.

The analysis presented in this paper provides a promising method for improving the robustness of adaptive systems. Some research could be done to study the effect of varying the time-window  $T$  in order to both capture the transient and provide sufficient adaptation. This paper used the stability margin notations for a single-input single-output (SISO) system, and the metrics-driven analysis was only performed for the pitch axis. However, further work and research needs to be pursued in order to perform frequency response for the multi-input multi-output (MIMO) systems, and in order to introduce the analysis of singular value for the metrics-driven adaptive control with BLSA method. The next step for this work is to apply the metrics-driven adaptive control method to a real-time flight control simulation software, and analyze the method using the full scale damaged generic transport model of aircraft.

It should be noted that there are other approaches that attempt to predict the time-delay margin of adaptive control systems. One such method is based on Lyapunov theory for a time delay system using Pade approximation.<sup>18</sup> However, the predicted time-delay margin using this approach is quite conservative and may not be well-suited for a practical implementation.

## VII. Conclusion

The application of bounded linear stability analysis on adaptive systems is evaluated in order to construct a methodology for establishing a metrics-driven learning paradigm that preserves margins during adaptation by adjusting the adaptive gain. With the purpose of achieving a certain phase margin requirement, the adaptive gain is re-calculated over a time-window during adaptation. Therefore, the adjusted adaptive gain is used in the adaptive law. The analysis is performed on a linear damaged twin-engine generic transport model. The analysis indicates that with the adjusted adaptive gain  $\Gamma$  the system robustness to uncertainty is increased significantly, and the desired phase margin criteria is achieved. By using the adjusted  $\Gamma$ , although the tracking performance is degraded to some point, the adaptive control signals in all axes become more stable. Future work will be conducted to further investigate the metrics-driven learning paradigm, and also to apply the method to realistic systems with different failure scenarios.

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